Name of the college – S. S. college J. Bad
Dept – Mathematics
Topic → Oblique Asymptotes of problem based on Algebraic curve
class – B.SC I (HONS)
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1.

Find the Asymptotes of the Curve.

$$(x+y)^2(x+2y+2) = x+3y+2$$

Solution

The Given Equation of the curve is

$$(x+y)^2(x+2y+2) = x+3y+2$$

it is of the form.

$$(x+2y+2)F_{n-1} + F_{n-2} = 0$$

∴ One of the Asymptote is

$$x+2y+2 = 0$$

and other two asymptotes are given by

$$x+y = \pm\sqrt{\lim_{x=-y\to\infty} \frac{x+3y+2}{x+2y+2}}$$

To evaluate this limit.

put $x = -\frac{1}{t}$ and $y = \frac{1}{t}$.

When $x = -y = \infty \Rightarrow t \to 0$

$$\therefore x+y = \pm \sqrt{\lim_{t\to 0} \frac{-\frac{1}{t} + \frac{3}{t} + 2}{-\frac{1}{t} + \frac{2}{t} + 2}}$$

$$= \pm \sqrt{\lim_{t\to 0} \frac{-1+3+2t}{-1+2+2t}}$$

$$= \pm \sqrt{\lim_{t\to 0} \frac{3+2t}{2t+1}}$$

$$= \pm \sqrt{\lim_{t\to 0} t\left[\frac{3}{t}+2\right]\frac{3}{1}} = \pm 2\sqrt{2}$$

Hence the Asymptotes are

$$x+2y+2=0$$

$$x+y-2\sqrt{2}=0$$

$$x+y+2\sqrt{2}=0$$

Problem: Find the Asymptotes of the Cubic curve

$$x^3+4x^2y+4xy^2+5x^2+15xy+10y^2-2y+1=0$$

Solution → Here Equation of the curve is

$$x^3+4x^2y+4xy^2+5x^2+15xy+10y^2-2y+1=0$$

$$\Rightarrow x\left(x^2+4xy+4y^2\right)+5\left[x^2+3xy+2y^2\right]-(2y-1)=0$$

$$\Rightarrow x[x+2y]^2+5(x+y)(x+2y)-2y+1=0$$

$$\Rightarrow [x+2y]^2+5(x+2y)\lim(1+y/x)+\lim\left(-\frac{2y}{x}+\frac{1}{x}\right)$$

When $x\to\infty$ $\frac{y}{x}\to -1/2$

$$\Rightarrow (x+2y)^2+5(x+2y)(1-1/2)+\left[-2(-1/2)+0\right]=0$$

$$\Rightarrow (x+2y)^2+\frac{5}{2}(x+2y)+1=0$$

$$\Rightarrow 2(x+2y)^2+5(x+2y)+2=0$$

i.e $2(x+2y)^2+4(x+2y)+(x+2y)+2=0$

$$\Rightarrow 2(x+2y)[x+2y+2]+(x+2y+2)=0$$

$$\Rightarrow [x+2y+2][2x+4y+1]=0$$

i.e $x+2y+2=0$

$2x+4y+1=0$

Since in this Cubic Equation of curve,
we find that x^3 is present but there is no term containing y^2

Hence, No Asymptote parallel to x-axis.

To Get the Asymptote parallel to y-axis

Co-efficient of $y^2 = 0$

$$4x + 10 = 0$$

$$2x + 5 = 0$$

Hence Required Asymptotes are

$$x + 2y + 2 = 0$$

$$2x + 4y + 1 = 0$$

$$2x + 5 = 0$$

Find the Asymptotes of the Curve.

$$3x^3 + 17x^2y + 21xy^2 - 9y^2 - 2ax^2 - 12axy - 18ay^2 - 3a^2x + a^2y = 0$$

Solution → Here the Equation of Curve is Cubic.
Therefore, we have three Asymptotes which are to be Found.